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Ethnomathematics of the Inkas

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Without Abstract

Under the shade of a tree some women are sitting. They are watching over several children, but at the same time their bodies are subtly swaying and their hands are busy moving threads. These women are weaving. As they talk among themselves, calculations are occurring: 40 × 2, 20 × 2, 10 × 2, etc. On their weaving tools symmetric patterns of geometric and animal figures are slowing emerging, produced from years of experience in counting and understanding symmetric properties. The procedures they follow have been instructed to them verbally as has been done for thousands of years, and they follow it precisely, almost subconsciously. In fact, these women are doing *mathematics*. They are calculating pairs of threads in blocks of tens (10, 20, and so on) and determining which colors of threads must go in which places so that half of emerging figures will be exactly copied across an axis of symmetry. These women, and likely some girls who are learning from them, are not writing down equations or scratching out the calculations on a notepad. Remarkably, the weaving is done from memory.

Weaving has existed in most cultures around the world, so the events and hence the mathematics in the previous paragraph could occur almost anywhere. In our case, we are going to consider the mathematics of the South American cultural group of the Quechua-speaking Inkas (Incas). As with weaving, we will see that Inka mathematics is intimately tangled with Inka culture.

Before we can jump into the details of their mathematics, we must take some time to understand who the Inkas were, and some small picture of what their culture was like. Backing up even further, the first term we must clarify is “Inka,” by which we refer to a collection of many groups who had a common government, religion and language, but were of distinct cultural origins. When we speak of the “Inka Empire,” we refer to the territory controlled by the Inka from about 1400 to 1560 (CE). The first Inkas started near Cuzco in present-day Peru and persistently moved on neighboring groups, forming an enormous empire that included part or all of Peru, Ecuador, Bolivia, Chile, Argentina, and southern Colombia (Fig. 1).

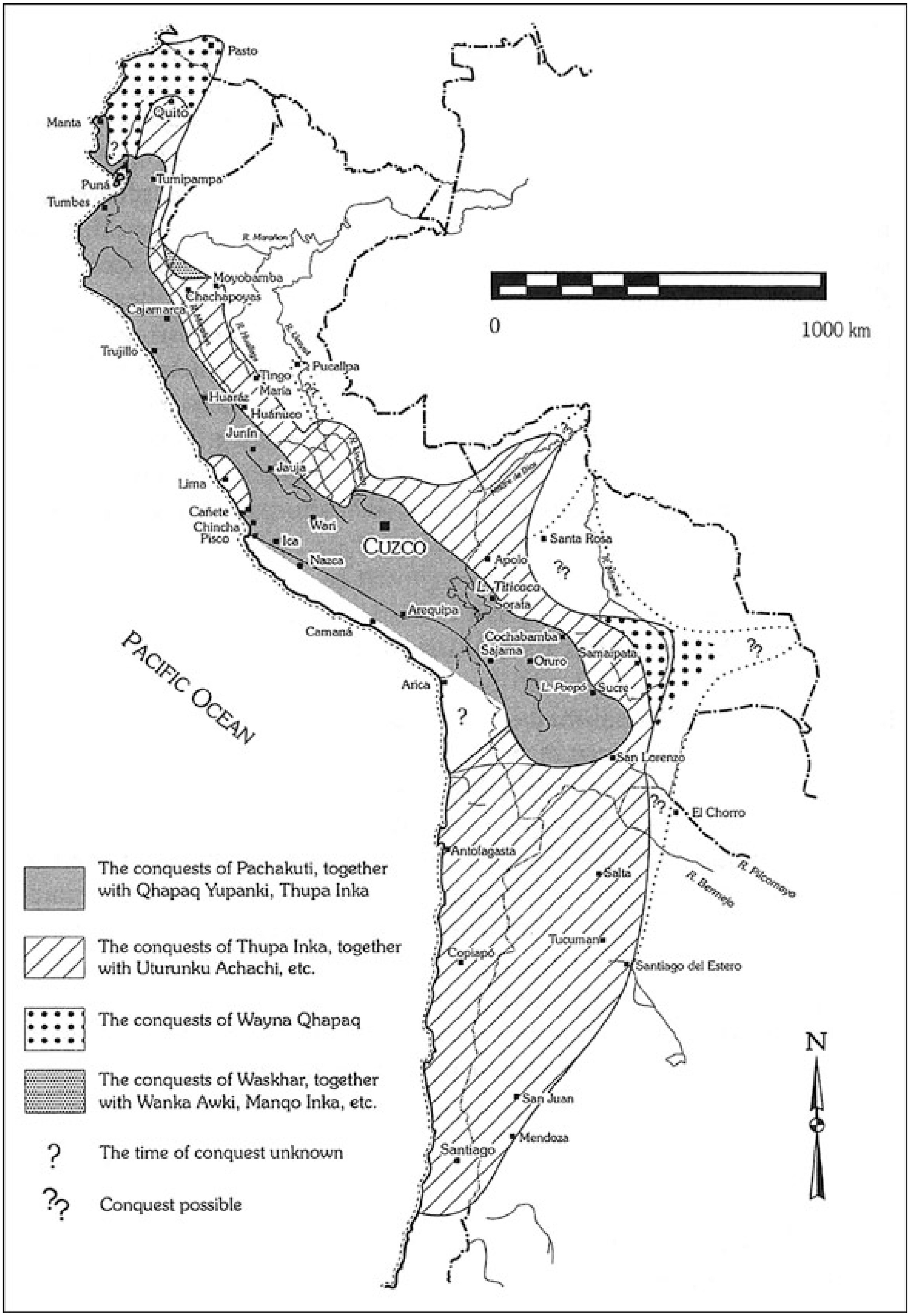

Fig. 1 The Inka Empire. From D'Altroy, *The Incas*. Oxford: Blackwell Publishing, 2003: 66. Used with their kind permission.

We have seen that there is mathematical thought occurring in weaving, but as we proceed, we can also observe factors such as geography that may have influenced the development of Inka mathematics.

Geographic, Climatic, and Environmental Aspects of Inka Mathematics

The Inka territory included regions of widely varying geography: coastal desert, high rugged Andes, the Lake Titicaca inter-mountain area, seacoast along the Pacific, and jungle on the eastern edge of the Andes as they descend into the vast Amazon basin. From pages 24–32 of D'Altroy (*2003*), we can get a glimpse of the situation, by noting that a 200 km trip by air from ocean to forest in central Peru would cross 20 of the world's 34 major life zones. In addition, the Andes range contains many areas that are rugged enough to be inaccessible even by horseback.

In terms of weather, there are numerous, unstable climate and geological patterns such as earthquakes, droughts, floods and the corresponding effects of periodic occurrences of el Niño.

These geographical aspects indicate several needs for mathematics. For example, all successful groups in the dry regions had to have some kind of effective water control in the form of irrigation and aqueducts. Next, those in the high altitudes had to have some form of flexible mountain agriculture, such as terrace farming. Moreover, in many parts of the Inka territory, the groups there had to construct bridges to cross deep canyons and difficult mountain areas. In effect, civil, and agricultural engineering were crucial elements of survival. Finally, as in almost any cultural group, knowledge of astronomy is important in terms of predicting planting and harvesting seasons, approximating weather changes, and general time keeping. Mathematics is, of course, necessary for all of these activities.

A Few Relevant Cultural Groups

As alluded to previously, there were many groups that formed what we are calling the Inkas, and we would like to mention just a few of them. The relevance is that these groups had their own mathematical concepts and practices, and as the Inkas absorbed these groups into their system, they almost certainly made use of some of that mathematical knowledge. A quite complete general reference on the various groups and their interactions with the Inka is in Moseley (*1992*).

Of the many competing cultural groups in what became Inka territory, we can first mention the Moche of the northern Peruvian coast, from about 100 to 700 CE. Later, in old Moche territory a substantial group called the Chimú arose, forming the Chimor Empire with its capital city of Chan Chan. The Moche-Chimú group was the largest Inka rival, and they inhabited the northern Peruvian coast during the Early Intermediate period to the Late Intermediate period (about CE 0–1470). They practiced river valley agriculture along several of the rivers that descend from the Andes to the Pacific, including the Moche River. They were not overcome by the Inkas until 1470.

Another group is the Huari of the central Andean highlands of Peru during the Early Intermediate and Middle Horizon periods (about CE 600–1000). The Huari engaged in advanced irrigated terrace farming and had quite mathematical artwork.

Next, we can mention the Aymara kingdoms of the Lake Titicaca region of the Late Intermediate period (about CE 1000–1400). The Aymara themselves consisted of several groups, e.g., the Colla and Lupaqa groups. Also, the Aymara had their own language group, and some dialects of Aymara are still spoken today. The Aymara also had extensive terrace agriculture as well as domesticated llamas and alpacas.

The last of this sampling of groups that we mention is the Nazca of the southern Peruvian coast of the Early Intermediate period (about BCE 400–CE 500). Some indication of Nazca understanding of geometry and astronomy can be seen in the lines and figures they created in the desert. It is still not clear what the lines represent, but there are connections between many of the lines and both astronomy and ritual of the Nazca. The interested reader can get some idea of this topic in the book edited by Aveni et al. (*1990*) and in the article on Nazca lines in this encyclopaedia. Of course, we cannot exit the Nazca topic without mentioning in passing the inspiration to mathematics teachers in the name of Maria Reiche (1903–1998). Reiche was a German-born teacher who made a daring career move in 1932 by moving to Peru. By the end of her career, she had played a crucial role in the preservation and study of the Nazca culture and lines. See her collected works in Reiche (*1993*) for more details.

All of the above groups had some knowledge of mathematical ideas. The *khipu* that Marcia Ascher will discuss in detail elsewhere in the encyclopedia existed long before the rise of the Inkas and was known for example, to the Huari and Aymara; see Chap. 10 of Quilter and Urton (*2002*). As the Inkas grew to control territory, their control and/or interaction with these groups must have played into their own mathematical understanding and development. We could even go much farther and say that this little piece on Inka ethnomathematics is only a peek into the larger realm of mathematical interaction, history, and culture of Andean societies, most of which has yet to be carefully researched.

Understanding Preconquest Inka Culture

Tahuantinsuyu, “Land of the Four Quarters,” is the word the Inka used to describe their territory. We would like to know a little about what Inka culture was like before the arrival of the Spanish in 1532. However, it turns out that it is not so easy to reconstruct an ancient culture.

First, although the Inkas were indisputably advanced in many regards, they did not use a writing system as we know it. In fact, the subject of how the Inkas created and maintained complexity in terms of society, government, and economy without communication through what we would consider a writing system is one of active debate. The khipu represents a mathematically based system of information keeping that could well have served in place of writing. Meanwhile, however, it is also the case that the Inka groups made use of oral tradition, whereby information, history, and social practices are passed along via oral descriptions. See Schneider (*1994*) for a general description of oral tradition. Also, an interesting description of Inka culture and literature can be seen in Lara (*1960*).

Next, there are problems with accuracy of information on the history of the Inkas. Because the only studies of Inka culture took place after the Spanish conquest, information about the Inkas is either substantially culturally biased, as in the case of most Spanish chroniclers, or is a study of a group that has changed significantly in nature, as is the case in studying present-day descendants of the Inkas. Thus, accurate information is difficult to obtain. For our purposes, two sources considered to be relatively accurate are those of Pedro Cieza de León and Felipe Guaman Poma de Ayala. In Ascher and Ascher (*1981*: 3), the reader may find a detailed description of original works and translations of Cieza de León's work, and in the bibliography here we have listed a reference to the works of Guaman Poma (1936). Guaman Poma is particularly known for the many drawings of Inka culture that he made, a few of which are presented in Fig. 19. From sources such as these, we can explain a few aspects of Inka culture that are relevant to our theme of mathematics.

One feature of the Inkas worth mentioning is that as they expanded into regions of other groups, they allowed those groups a certain amount of local control. There are two advantages to this attitude. One is that the subjugated groups would not be as likely to reject Inka rule. The other, relevant to our theme of mathematics, is that by allowing the conquered groups to retain some original culture, the Inkas could use and improve mathematical and engineering ideas of those groups, such as astronomy and agriculture. This last comment implies that some of the mathematics of the Inkas was probably diffused from other groups.

Also relevant to the development of mathematics in the Inka region is that of economy. Starting with very early organized groups and extending even to the present, trade has been an important factor in the societies in the Inka region. In addition, the Inka economy included an extensive taxation system. Later, we will see that this taxation, among other activities, could be recorded on the Inkas' khipus. A good reference to the ideas of trade and interaction of the various groups can be found in Moseley (*1992*).

Quechua Number Words

Our first mathematically close look at the Inkas comes by way of discussing Quechua number words, which reflect Inka numeration. In a study of numbers in cultural mathematics, there are two distinct aspects that we must consider: Number words and number symbols (representation). These two concepts are not the same.

The first things we must clarify have to do with Quechua, the language used by the Inka. Until the 1960s, it was commonly thought that Quechua originated and spread with Inka expansion. In Mannheim (*1985*), Mannheim (*1991*), Stark (*1985*), Urton (*1997*), and Weber (*1989*), the reader may find explanations and references indicating that Quechua in fact originated in northern central Peru and later split into essentially two branches. For our intentions, we take as our model Weber's description of Huallaga (Huánuco) Quechua, which is often referred to as *Quechua I* or *Quechua B* in the references. As Weber indicates on, Huallaga Quechua seems to have suffered fewer changes than some other Quechua dialects, which implies that our discussion here of number words should have some accuracy.

Here is a short list of number words from Huallaga Quechua:

huk – 1, *ishkay* – 2, *kimsa* – 3, *chuska* – 4, *pichqa* – 5, *soqta* – 6, *qanchis* – 7, *pusaq* – 8, *isqon* – 9; *chunka* – 10, *pachak* – 100, *waranqa* – 1000.

The format of more complex number words is the following:

[multiplier] {nucleus} (adder).

The nucleus is always a power of ten. For the examples that follow below, we will employ the above notation, namely, multipliers – [], nuclei – {} and adders – ().

Example 1: *isqon pachak*: [9] {100} = 9 ¥ 100 = 900.

Example 2: *qanchis chunka pichqa*: [7] {10} + (5) = 75

Example 3: 347,002: [[3] {100} ([4] {10} (7))] {1000} (2).

kimsa pachak chuska chunka qanchis waranqa ishkay.

From the number words and the examples above, we can deduce that the Inkas counted in base ten; i.e., they used a decimal system.

Looking Deeper: Weaving, Symmetry, and Counting

The previous section describes some basic facts about number words of the Inkas. However, that information tells us little about the cultural context of Inka mathematics. Such contexts between mathematics and culture pull us into the field of ethnomathematics. Good places to start in the study of ethnomathematics in general are with Selin (*2000*), Ascher, (*2002*), Closs (*1986*), Pacheco (1998), Urton (*1997*), and Zaslavsky (*1973*).

What comes out of ethnomathematical considerations is the understanding of how a particular culture perceives and interprets mathematics. It is often surprising to learn that some cultural activity of a particular group involves elegant mathematics that are not easy to see until we look closely and think differently. Moreover, although modern Western mathematical notation of the mathematics being used may not make sense to people of the group, it becomes clear that they nevertheless understand the concepts. The interesting part is that the Western and non-Western interpretations can be dramatically different. We will next examine some such activities for the case of the Quechua-speaking Inkas.

Let us look more carefully at the situation of the women who were weaving, described at the beginning. The first mathematical observation we can make is to notice the symmetry patterns that evolve from the weaving process. In chapter six of Ascher (*1991*), we find a careful discussion of decoration and various translations, reflections and rotations that reveal a cultural emphasis of the Inkas on formality, precision, and repetition. Fig. 2a–d show some examples of weaving patterns of the Inka and some other cultural groups eventually controlled by the Inkas. In Fig. 2a for example, we can observe many types of symmetry. One such symmetry is horizontal: Imagine a horizontal line drawn halfway through the figure. Then we see that the pattern in the upper half is exactly repeated in the lower half. Similarly, if we draw a vertical line down the middle of the figure, then the pattern on the left half is exactly repeated on the right half, so the figure has vertical symmetry. Now let us look at Fig. 2b. Except for slight displacements of some of the smaller parts of the figure, it has horizontal symmetry. It does not have vertical symmetry, however: Observe that if we draw a vertical line through the middle of the rectangular spirals part of the figure and reflect the figure across that line, then the spirals will appear in a different orientation; that is, we will not see the same pattern repeated. On the other hand, the rectangular spirals pattern appears again in Fig. 2c, which although it does not have vertical symmetry, has rotational symmetry: Imagine putting a tack in the middle of the strip pattern and rotating the figure 180°. The rectangular spirals then appear in the same orientation as in the original unrotated figure. Now let us look at ceramics, namely the item marked “300” from Fig. 3. It has several symmetry properties including vertical, horizontal, and rotational symmetry. On the other hand, item 311 of Fig. 3 only has vertical and translational symmetry (translational symmetry means the pattern is repeated if we slide the figure to the right or to the left). Next, item “401” of Fig. 4 only has translational symmetry.

Fig. 2 (a) Bolivian textile pattern. From Tamara E. Wasserman and Jonathan S. Hill, *Bolivian Indian Textiles, Traditional Designs and Costumes*. New York: Dover Publications, 1981, Plate 11. (b) Bolivian textile pattern. From Tamara E. Wasserman and Jonathan S. Hill, *Bolivian Indian Textiles, Traditional Designs and Costumes*. New York: Dover Publications, 1981, Plate 17. (c) Bolivian textile pattern. From Tamara E. Wasserman and Jonathan S. Hill, *Bolivian Indian Textiles, Traditional Designs and Costumes*. New York: Dover Publications, 1981, Plate 17. (d) Bolivian textile pattern. From Tamara E. Wasserman and Jonathan S. Hill, *Bolivian Indian Textiles, Traditional Designs and Costumes*. New York: Dover Publications, 1981, Plate 13.

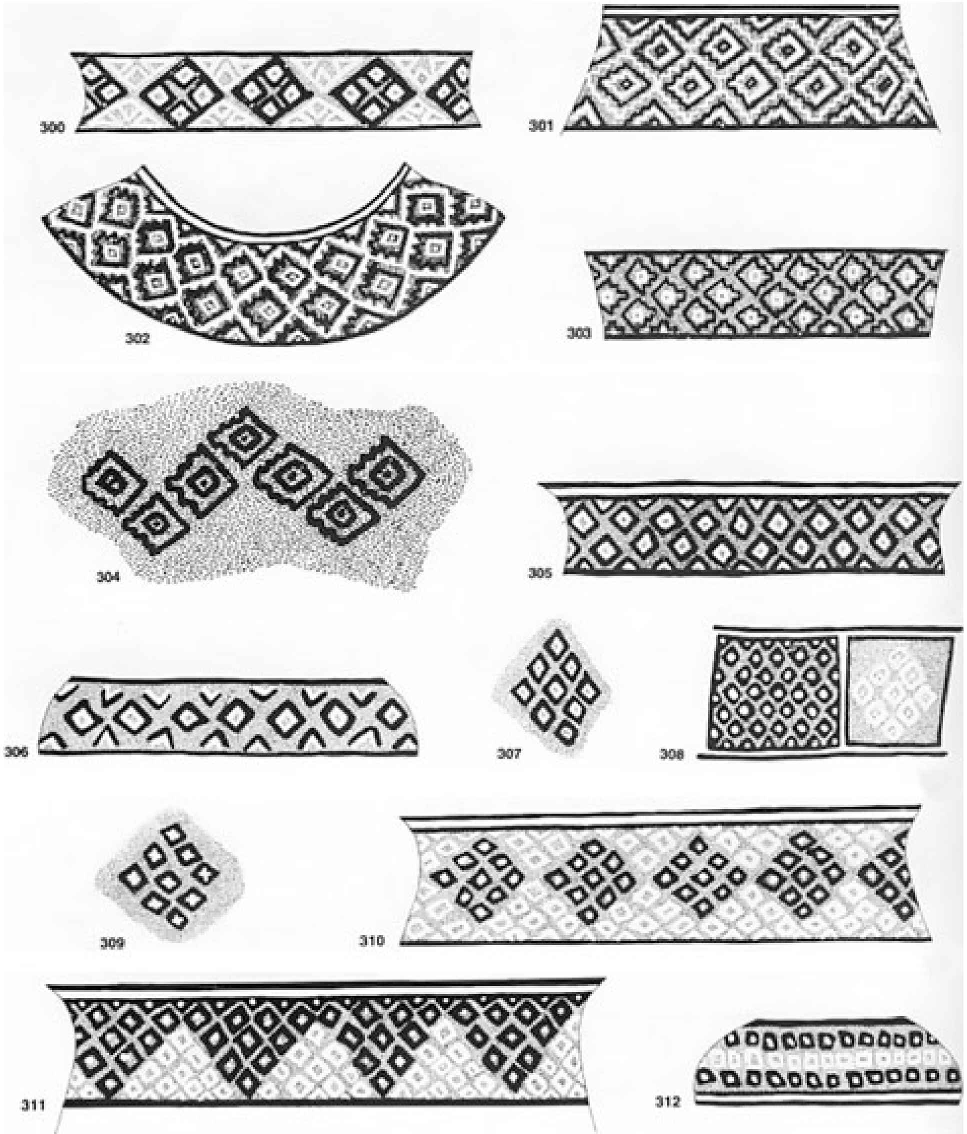

Fig. 3 Ica design patterns. From Dorothy Menzel, *Pottery Style and Society in Ancient Peru*. Berkeley: University of California Press, 1976, Plate 26. Used with their kind permission.

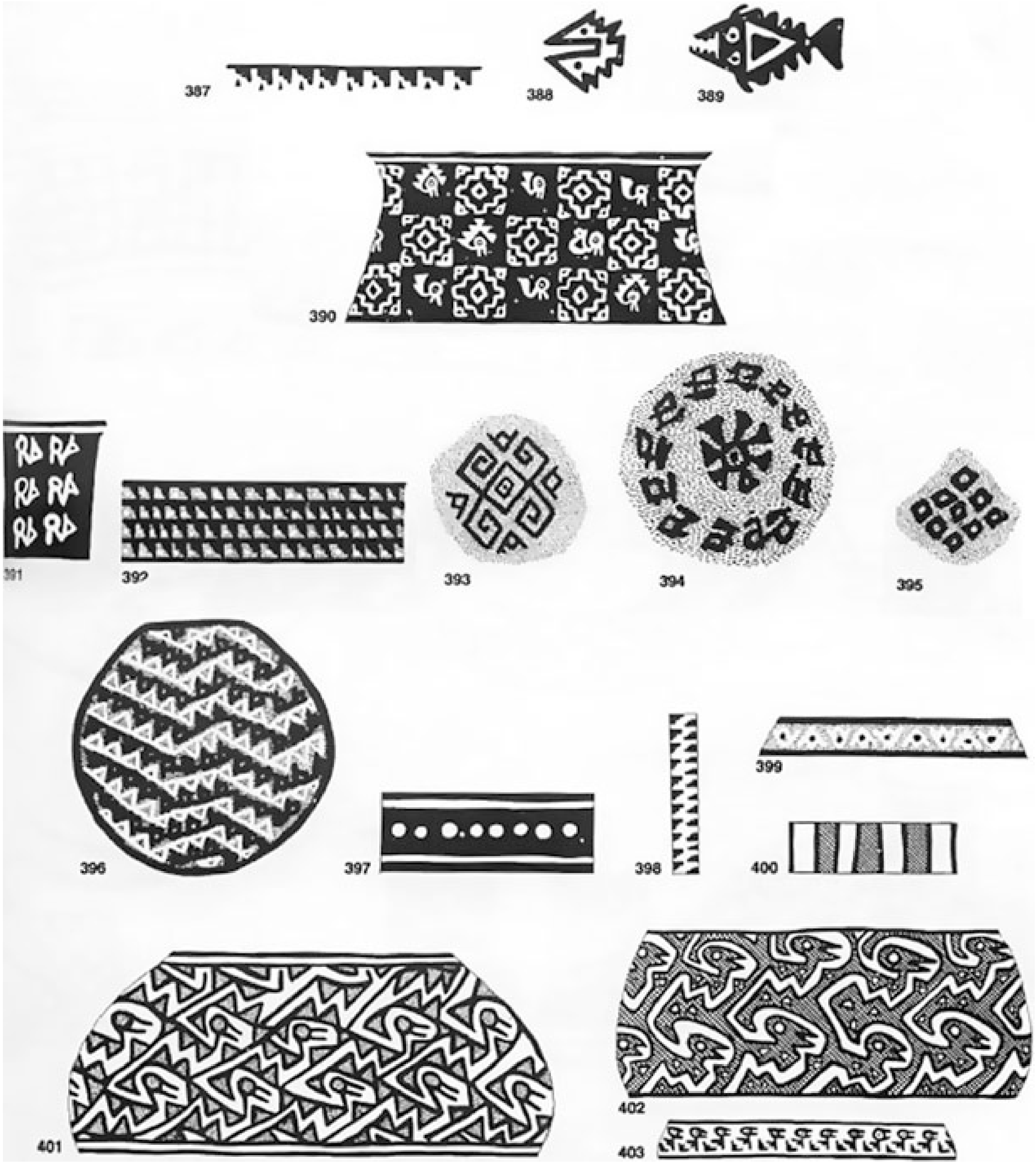

Fig. 4 Ica design patterns. From Dorothy Menzel, *Pottery Style and Society in Ancient Peru*. Berkeley: University of California Press, 1976, Plate 31. Used with their kind permission.

Furthermore, symmetry can appear in contexts other than weaving. Go get yourself a snack or something refreshing to drink while you ponder the symmetry properties in Figs. 3–5. If you want to look even deeper into more types of symmetry properties, you can read Ascher (*1991*), Chapt. 6.

Fig. 5 Ica design patterns. From Dorothy Menzel, *Pottery Style and Society in Ancient Peru*. Berkeley: University of California Press, 1976, Plate 45. Used with their kind permission.

The above is good practice for learning about symmetry patterns in general, but we must get back to the cultural part of this. We can ask questions like: What can we deduce from the symmetry properties used in a particular cultural group? In the case of the Inkas, we see the repeated preference for a lot of symmetry. This preference indicates a strong sense of order and precision in Inka culture and mathematics. This corresponds with our general knowledge of the Inkas as being a culture that kept careful records via their *khipus*.

Here is another question: how do the weavers know how to construct the patterns in order that they have such symmetry? Even in the case of what at first appears to be a simple shawl having several stripes of varying widths, we can ask as to how the weaver knew how to obtain the widths from the threads. Look at Fig. 2a again. In these questions we are in fact asking about mathematics. The people (mainly women) who weave such items must in fact count (*yupa*: "to count or account") threads. For the case of stripes, the number of say, red threads for the first stripe must correspond in some way to the number of red threads that might appear later in the pattern, so that the two red stripes have the same width. The numbers of threads to be counted in each step of a weaving process is often complicated, and must be decided upon ahead of time. Indeed, the weaver must know in advance what the entire pattern will look like. Once the process starts, what is remarkable is that the weaver does the counting of threads mentally. A detailed discussion of how groups of threads are counted by Inka (Quechua) weavers can be found in Chapter four of Urton (*1997*). In our context of ethnomathematics, we can observe some mathematical properties in Inka weaving. One such property described in Urton (*1997*) (see Fig. 6) is that thread counts are usually done in blocks of 10 and 20, and that in general what is being counted are values of pairs of numbers of threads. Notice the influence of the decimal system. The complex geometric or animal motifs are created by a process called *pallay*- “to pick up.”

It is worth mentioning that the master weavers or *Mamas* are women who most likely started weaving when they were girls and reached a high level of expertise. They were treated with special respect. Their abilities in counting, understanding patterns of symmetry, and in geometry were part of that expertise. The ethnomathematical aspect of this situation is this: if we asked one of these women to explain geometric or symmetry properties in terms of lines, rotations, polygons, and so forth, they probably would not be able to explain them. Yet, they clearly understand these mathematical concepts; the difference is that their understanding comes from the perspective of a weaver who must create a pattern.

A Bit More About Counting, Pairs, and Even Numbers

In Quechua culture, counting is closely tied to important cultural aspects. Kinship and social organization, as explained by Urton (*1997*), represent one such aspect. On page 85 for example, he tells us that ears of corn on a stalk are carefully named in the order in which they appear, with the first representing the “mother” and others being defined in accordance to the first as offspring of the mother (Fig. 7).

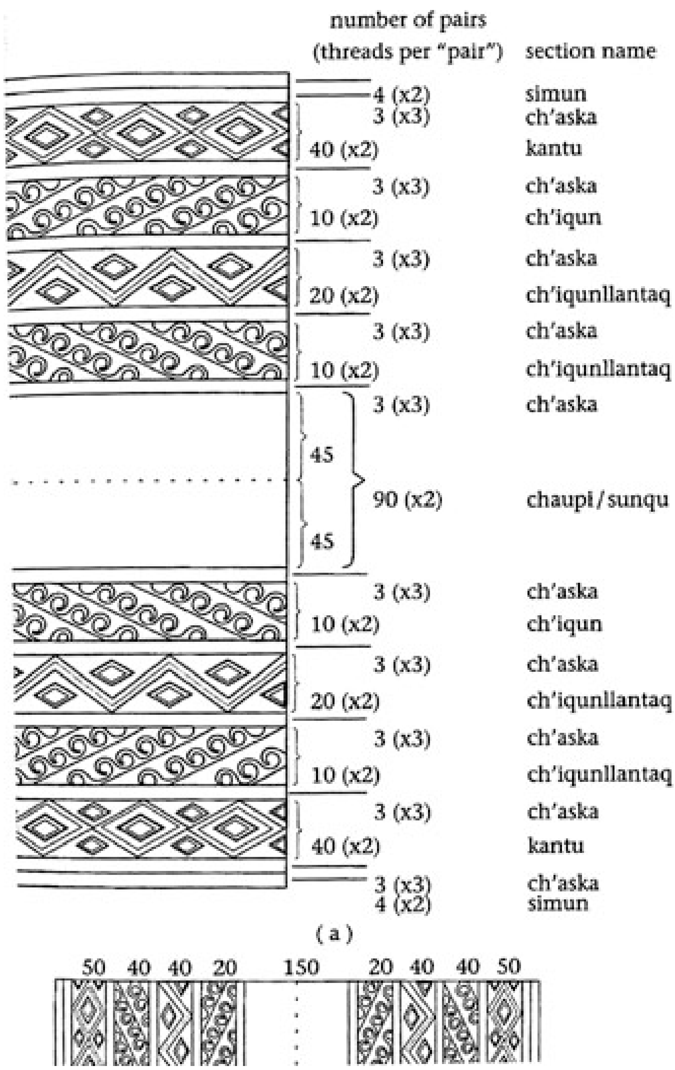

Fig. 6 Warp counts on two axsus from Candelaria. From Fig. 4.6, pg. 121 of the *Social Life of Numbers: A Quechua*

Ontology of Numbers and Philosophy of Arithmetic, by Gary Urton with the collaboration of Primitivo Nina Llanos, @1997. By permission of the University of Texas Press.

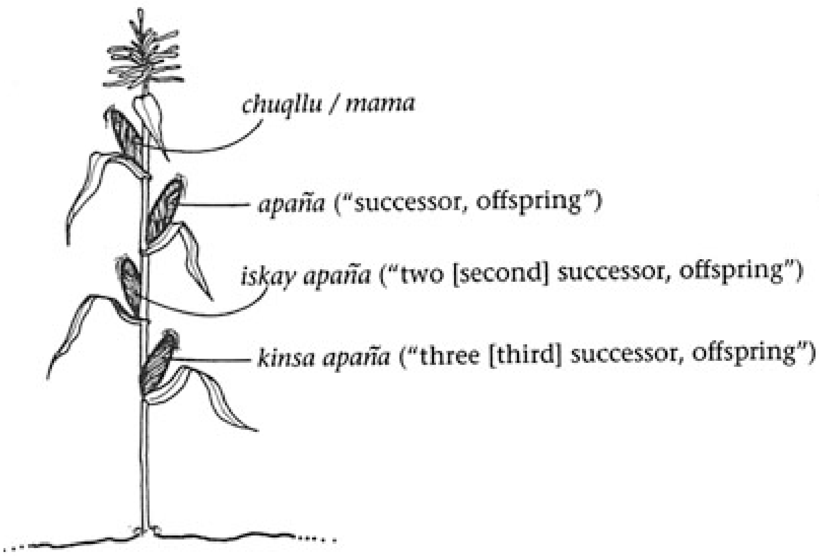

Fig. 7 The 'birth order' and naming of ears of corn. From Figure 3.3, pg. 86 of the *Social Life of Numbers: A Quechua Ontology of Numbers and Philosophy of Arithmetic*, by Gary Urton with the collaboration of Primitivo Nina Llanos, @ 1997. By Permission of the University of Texas Press.

Note that the order of the counting is as important as the counting itself. As for the subject of pairs, it turns out that the Inkas view the property of being even in a count as extremely important. In fact, we could go so far as to say that odd numbers are considered as incomplete pairs. We saw above that the counting of threads in the weaving process is done in pairs of threads. There are many more instances in which one counts pairs in Quechua society. The perception of even versus odd numbers is in fact one of the main themes of Urton's (*1997*) book. The cultural impetus for this viewpoint is that the Inkas constantly strive to have proper order in their world. Such order even extends to contexts like social situations. If a person has done something inappropriate, then something must be done by him or by the community so that proper social order is regained. This can be thought of as completing an incomplete pair. By the way, notice that this persistence of order and proper organization appeared in the symmetry patterns, too.

A Few Remarks About Astronomy

Inkan astronomers knew about the solar, lunar, and Venusian cycles. They had a solar calendar and festival calendar based on 12 lunar cycles, plus adjustments to account for the difference necessary to make a full 365 day year.

Throughout the city of Cuzco there are remains of some of about 400 markers, called *huacas,* along imaginary lines called *ceques* (also *zeq'e*). These lines originate from the center of Cuzco. Figs. 8 and 9 show some details of the system, though you will have to read Chapt. 7 of D'Altroy (2002) to fully understand the figures. These huacas had ritual and social significance. Some had connections with astronomy, hence mathematics. In particular, the Inka used several of the ceque lines as part of astronomical observations such as the June solstice. Furthermore, the ceques and huacas were crucial components of the Inkan religious ceremonies and in fact the huacas are more accurately described as sacred shrines. For more details about Inkan astronomy and the ceque lines, see for example, Moseley (*1992*), pages 78–79, Chapt. 7 of D'Altroy (2002), or Urton (*1981*), and the article on Quipus in this encyclopaedia.

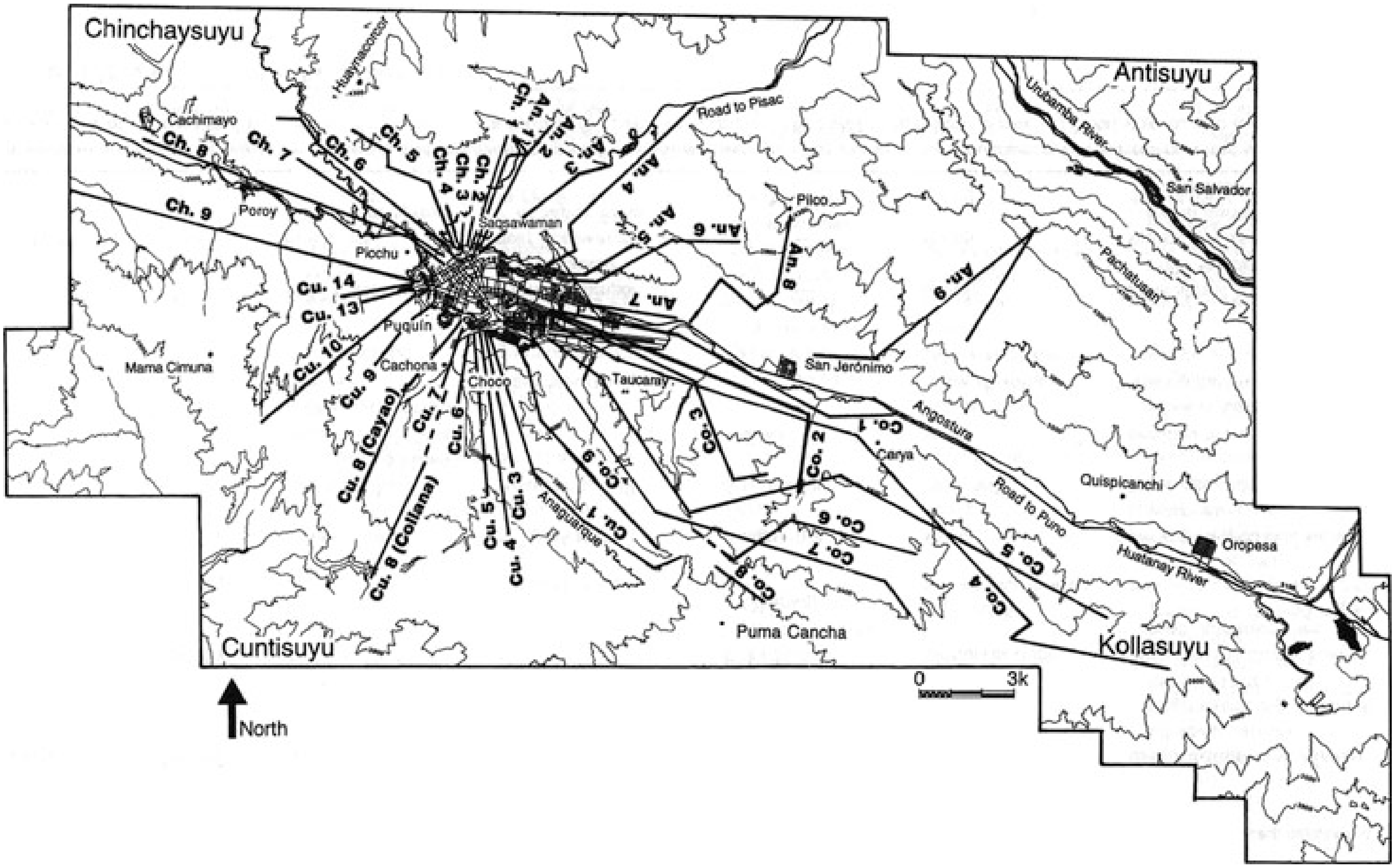

Fig. 8 Cuzco's ceque system. From Terence D'Altroy, *The Incas*. Oxford: Blackwell Publishing, 2003. Fig. 7.1, page 160. Used with their kind permission.

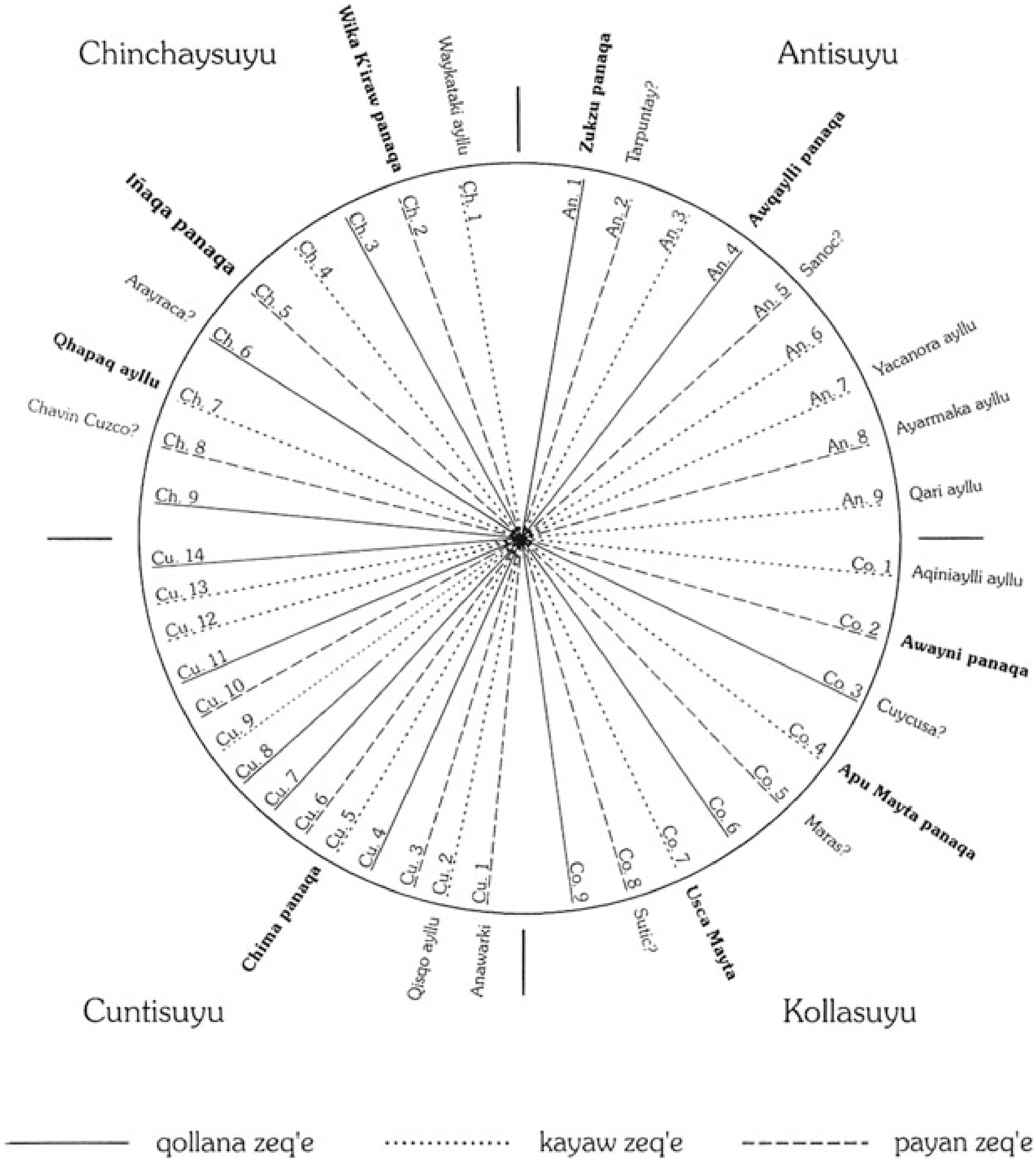

Fig. 9 Schematic diagram of Cuzco's ceque system. From Terence D'Altroy, *The Incas*. Oxford: Blackwell Publishing, 2003. Figure 7.2, page 161. Used with their kind permission.

Astronomy and culture quickly become intimately involved with each other. We know that astronomy and mathematics also have a close relationship, so we can conclude here that astronomy, culture, and mathematics of the Inkas are intertwined.

Yupana

The Inka performed rather complicated computations. The khipu was a record keeping device not used for calculating. In fact, most of the knots on known khipus are tight, implying that the values on them were usually intended to be fixed. This leads us to ask the question: How did they make the computations? The answer to this question is still not completely understood. One possibility arises in the first of the drawings of khipus made by Guaman Poma. (See Ascher's article on the Quipu.)

The rectangular grid of solid and unfilled dots appears to be a kind of counting board; however, Guaman Poma gives no explanation of the grid. Certainly there would be reason to believe that the grid has mathematical meaning because of its appearance with a khipu. A curious aspect of the grid is that each row contains 11 dots, something that obviously does not coincide with a decimal counting system. In addition, because Guaman Poma's work occurred after the Spanish conquest of Peru, the grid does not necessarily represent a device of the Inka.

On the other hand, proposals of mathematical uses of the grid have been given, with the term *yupana* (Quechua for the verb to count) used to denote it, such as appears in Mackey et al (*1990*), Higuera (*1994*), and Burns Glynn (*1981*, *1990*).

The mathematical methods for counting that are presented make logical sense, such as filling the 11th dot as a placeholders for powers of ten. Included are photos of three-level rectangular stone figures that resemble counting grids in some ways that could have served as abaci for counting. There are descriptions by Spanish chroniclers of observations of Inkas counting on these rectangular forms, but here we encounter the problem of accuracy. A conclusive explanation of the grids and rectangular forms still has not been given. However the works cited here point toward possible patterns of counting or symbols.

See also: Nazca Lines, Environment and Nature in the Andes

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