# Mathematics of the Hñähñu: the Otomies

Thomas E. Gilsdorf
Department of Mathematics
University of North Dakota
101 Cornell Street, Mail Stop 8376
Grand Forks, ND 58202-8376

thomas.gilsdorf@und.nodak.edu

**Abstract**:

*English*: This article is about the mathematics of the cultural group known as the Otomies, of present day central Mexico. In particular, I discuss the Otomi number system and a comparison of that system with Aztec counting, Otomi art and decoration, mathematical symbols that appear in some Mesoamerican codices, and the Otomi calendar.

*Español*: Este artículo es del tema de matemáticas culturales de los Otomíes, de la región central de lo que hoy en día es México. En particular, describo el sistema numérico de los Otomíes y una comparación con el sistema numérico de los Aztecas, el arte de los Otomíes, los símbolos matemáticos que aparecen en unos códices mesoamericanos, y el calendario otomí.

## 1. *Who are the Hñähñu*?

The cultural group we are going to consider is generally referred to as the Otomies (pronounced "oh- toh- MEES"). The name Hñähñu ("hñ" is pronounced like the "ny" in "canyon", preceded by a nasalized "h") is one of the names by which people of this culture refer to themselves. In general, people of Hñähñu culture use a variant of this word when referring to themselves with other members of the same culture, but use the word *Otomi* in communications in Spanish or with people from outside the culture. The term *Otomi* has been used in a derogatory way since before the arrival of the Spanish. Nevertheless, there is some consensus among anthropologists who study *Hñähñu* culture that it is better to use the word *Otomi*, presumably in a positive way, so as to work toward creating a more positive image of the people who are often referred to with this word. A discussion of the meanings of the terms *Hñähñu* and *Otomi*, and the negative ways in which the Otomies have been portrayed over the years can be found in (Wright, 2005; p. 19) and (Wright, 1997). I will use the term *Otomi*, and hopefully this work will serve as a positive description of the people, as viewed through an analysis of their number system, weaving and embroidery, and calendrics- the main themes of this article.

The Otomies are an indigenous group of what is now central Mexico. The modern- day territory in which significant numbers of Otomies live and practice some form of their traditional culture includes large parts of the Mexican states of Hidalgo and Mexico, with smaller areas in the states of Puebla, Guanajuato, Michoacán and Veracruz. There is also a significant number of Otomies in present day Mexico City (the Federal District). Over the last several centuries the Otomies have migrated because of issues such as conflicts with the Aztecs[1] and the arrival of the Spanish in 1521.

The Otomies represent one of the largest and oldest indigenous populations of what is now Mexico. According to Lastra (2006; pg. 25), the number of speakers of Otomi (including those who also speak Spanish) as of 2000 was approximately 292,000. This gives a rough estimate of the number of people who practice Otomi culture to some extent. Wright (1997; pg 439 – 441), shows that there is significant evidence to conclude that the Otomies represent one of the oldest cultural groups of what is now central Mexico. Moreover, the Otomi language is part of the Otomangue family (Lastra, 2006 or Soustelle, 1993 [1937]), which is neither Yuto Aztecan nor Mayan. Thus, the Otomies represent a very old and very large cultural group, and one that is distinct from the more well- known Aztec and Mayan cultures.

A consideration that is pervasive in the study of Otomi culture is the interaction between the Otomies and the Aztecs. The history of Aztec– Otomi interaction and conflict is long and complicated[2], and one consequence of that history is that it is often difficult to distinguish between cultural aspects of these two groups. Understanding the mathematics of the Otomies requires understanding some basic facts about the mathematics of the Aztecs, and we will encounter this issue several times. Our first topic is the Otomi number system.

**2.** *Numeration*.

In this section, I will give a list of Otomi number terms and indicate the mathematical constructions that go with it. What follows is a comparison with Aztec number words, and conclusions that come from that comparison. The Otomi number words in Table 1 below come from the dialect of Otomi spoken in the Mezquital Valley of the state of Hidalgo. I have taken the information for these number words, including the linguistic notation that appears, from

[1] See pages 3- 4 and Note 2, pg. 294 of (Smith, 2003) regarding the use of the word "Aztec" versus "Mexica". Though both terms are widely used, I have chosen to use "Aztec" here.
[2] See (Soustelle, 1993 [1937]), pages 463 – 467.

(Barriga Puente, 1998; p. 229), originally referenced in (Merrifield, 1968). See also (Acuña, 1990) and (Thomas, 1897). A comparison of Otomi dialects in Barriga Puente (linguistic category 49) shows that the Mezquital Valley dialect has had less influence from Spanish than most of the other Otomi dialects.

| **Number** | **Otomi expression** |
|---|---|
| 1 | *ˀna* |
| 2 | *yoho* |
| 3 | *hñu* |
| 4 | *goho* |
| 5 | *kïtˀa* |
| 6 | *ˀrato* |
| 7 | *yoto* |
| 8 | *hñato* |
| 9 | *gïto* |
| 10 | *ˀrætˀa* |
| 11 | *ˀrætˀa ma- ˀra* |
| 12 | *ˀrætˀa ma- yoho* |
| 20 | *ˀnate* |
| 39 | *ˀnate ma- ˀrætˀa ma- gïto* |
| 63 | *hñu ˀrate ma- hñu* |
| 85 | *goho ˀrate ma- kïtˀa* |
| 100 | *ˀna nthebe* |
| 1000 | *ˀna ˀmo* |

Table 1. *Otomi number words.*

For the numbers from one to five, the only connection between terms that appears is the possibility that *goho* for four might represent twice two. On the other hand, starting at the term for six, there begins to be a pattern. Notice that the word for six, *ˀrato*, can be split into the parts *ˀra* and *to*. The part *ˀra* is similar to *ˀna* from the number one, but it is a good idea to look further down the list before concluding anything. The word for seven, *yoto*, splits into *yo* and *to*, and the word for eight, *hñato* splits into *hña* and *to*. From these examples, we start to see part of the word for two in the word for seven, part of the word for three appears in the word for eight, and part of the word for four appears in the word for nine. The word for ten no longer follows the pattern, however the word for eleven appears to represent the construction of "ten plus one". We can now conclude that the $^{?}$*ra* in the term for six is probably "one", and that *to* means "to add to five". Hence, six is "one plus five", seven is "two plus five", eight is "three plus five", and nine is "four plus five".

Let us now look again at the expression for eleven, $^{?}$*ræt*$^{?}$*a ma*- $^{?}$*ra*. It is easy to deduce that "11 = 10 + 1", and similarly, "12 = 10 + 2". It turns out that this pattern continues until twenty, when a new word appears for that value. Table 2 shows the constructions of the values shown in Table 1, with an added number, 2488.

| Number | Otomi phrase | Construction |
|---|---|---|
| 1 | *ʔna* | |
| 2 | *yoho* | |
| 3 | *hñu* | |
| 4 | *goho* | |
| 5 | *kïtʔa* | |
| 6 | *ʔrato* | 1 + 5 |
| 7 | *yoto* | 2 + 5 |
| 8 | *hñato* | 3 + 5 |
| 9 | *gïto* | 4 + 5 |
| 10 | *ʔrætʔa* | |
| 11 | *ʔrætʔa ma- ʔra* | 10 + 1 |
| 12 | *ʔrætʔa ma- yoho* | 10 + 2 |
| 20 | *ʔnate* | |
| 39 | *ʔnate ma- ʔrætʔa ma- gïto* | 20 + 10 + 4 + 5 = 20 + 10 + 9 |
| 63 | *hñu ʔrate ma- hñu* | 3 x 20 + 3 |
| 85 | *goho ʔrate ma- kïtʔa* | 4 x 20 + 5 |
| 100 | *ʔna nthebe* | 1 x 100 |
| 1000 | *ʔna ʔmo* | 1 x 1000 |
| 2488 | *yoho ʔmo ne- goho nthebe ne- goho ʔrate ma- hñato* | 2 x 1000 + 4 x 100 + 4 x 20 + 3 +5 = 2 x 1000 + 4 x 100 + 4 x 20 + 8 |

Table 2. *Otomi number expressions and structures.*

These constructions of Otomi numbers have been known for some time. Specific details of this counting system can be found in (Barriga Puente, 1998; pg. 229, Table 49ch). Nevertheless, we can ask ourselves here, “What is the counting base for the Otomi number system?” Indeed, there are values based on five (for numbers 6 to 9), while other values are constructed using ten and twenty. So, our conclusion is that Otomi counting is not done by a single base but rather by a combination of bases. In particular, the Otomi system would be called a 5 - 10 - 20 system; see (Closs, 1986; pg. 3). The Otomi system follows the counting patterns of several Mesoamerican cultural groups. For example, the Aztecs use[3] a similar 5 - 10 - 20 system; see for example, (Payne and Closs, 1986; pg. 214 – 218). The Mayans use a 10 - 20 system according to (Closs, 1986; pg. 292 – 293). Examples of counting systems of other Mesoamerican groups can be found in (Barriga Puente, 1998).

I will now compare the Otomi system with Aztec counting system. Because the structure of (counting) words is a linguistic consideration, the Aztec words are described in terms of their language, Náhuatl (pronounced "NAH- wah- tl"). Here is a list of Náhuatl numbers words, as described in (Payne and Closs, 1986; pg. 214 – 218).

| **Number** | **Náhuatl phrase** | **Construction** |
|---|---|---|
| 1 | *ce* | |
| 2 | *ome* | |
| 3 | *yei* | |
| 4 | *nahui* | |
| 5 | *macuilli* | |
| 6 | *chicuace* | 1 + 5 |
| 7 | *chicome* | 2 + 5 |
| 8 | *chicuei* | 3 + 5 |
| 9 | *chiconahui* | 4 + 5 |
| 10 | *matlactli* | |
| 11 | *matlactli once* | 10 + 1 |
| 12 | *matlactli omome* | 10 + 2 |
| 13 | *matlactli omei* | 10 + 3 |
| 15 | *caxtolli* | |
| 16 | *caxtolli once* | 15 + 1 |
| 17 | *caxtolli omome* | 15 + 2 |
| 20 | *cempoalli* | |
| 30 | *cempoalli ommatlactli* | 20 + 10 |
| 37 | *cempoalli oncaxtolli omome* | 20 + 15 + 2 = 20 + 17 |
| 60 | *eipoalli* | 3 x 20 |
| 100 | *macuilpoalli* | 5 x 20 |
| 400 | *tzontli* | |

Table 3. *Náhuatl number words.*

[3] I use the present tense in describing numerical expressions in Otomi, Náhuatl, and Mayan as a reminder that there are large numbers of people who still speak the languages of those groups.

The first similarity between the Otomi and Aztec number systems we encounter is that the structures for the expressions for six to nine are the same. This is worth looking into deeper.

It is generally understood that the Aztecs are a Yuto- Aztecan speaking group that migrated to present day Mexico City from somewhere north of that location. As indicated in the introduction, the Otomies are a very old Mesoamerican culture, and were already established in central Mexico when the Aztecs arrived. Therefore, it makes sense to examine maps of linguistic groups. In (Valiñas Coalla, 2000), there are several tables and maps of 16th Century linguistic information regarding number words of Northern Yuto- Aztecan and of Southern Yuto- Aztecan. The geographical area he studies includes the present day southwestern United States and northwestern Mexico. One such group from the northern part of this geographic area is the Opata ("oh PAH tah"). The following Opata number expressions are taken from (Valiñas Coalla, 2000). The expressions for one to ten are: 1- *seni*, 2- *gode*, 3- *vaide*, 4- *nago*, 5- *marizi*, 6- *bussani*, 7- *seni- bussani*, 8- *go nago*, 9- *kimakoi*, 10- *makoi*. Now, notice that the word for six has no connection with the word for five. The expression for seven is: "7 = 1 + 6". For eight we have: "8 = 2 x 4", and nine is "9 = 10 – 1". A look through the other Yuto- Aztecan dialects from the northern part of the region (roughly the part that would be in the present day southwest of the U.S.), shows that most such dialects follow a pattern similar to the Opata. Meanwhile, checking the dialects that are further south, we see that most of those Yuto- Aztecan dialects have the same structures for the numbers six through nine as do cultural groups from central (present day) Mexico, including the Otomies. Several cultural groups of Yuto- Aztecan speakers migrated toward central Mexico over the years before the rise of the Aztecs (Smith, 2003; p. 37). It appears that groups that migrated closer to central Mexico adopted number expressions similar to those of groups such as the Otomies. Based on these observations, we can conclude that the structures of the Aztec number expressions for six to nine represents an influence by existing cultural groups of central Mexico, including the Otomies. This conclusion is supported by Doris Bartholemew (see Bartholemew, 2000), in which she concludes that the Aztec expressions for six to nine were directly influenced by the Otomies. The difference between her analysis and ours is that hers is based on anthropological considerations, while ours is based on considering mathematical structures.

**3.** *Weaving and embroidery*.

The Otomies have traditionally been highly regarded for their skills at textile arts, and I would like to examine the mathematics of Otomi weaving and embroidery. Some general observations about weaving and embroidery will motivate the mathematical considerations. I will focus on the Otomies, though many of my statements hold for traditional textile artists in general. Also, I will tend to refer to Otomi textile artists as female because most are women, though there are male weavers and embroiderers.

First, to become a traditional Otomi weaver is a life long process. A common situation is one in which there are several generations involved in creating woven or embroidered projects. That is, there is often a gathering that includes girls, mothers, aunts, grandmothers, and so on. Second, Otomi weavers and embroiderers must keep track of many counts of threads, and must make precise measurements. They must know the entire design from memory. The impressive part of the weaving or embroidery process is that the artist typically is not using diagrams for the patterns, nor is that person using a ruler to measure distances[4]. A third consideration is that the products finished by the artists (as is the case in many traditional contexts) often have important cultural significance. Finally, it is important to mention that many kinds of weaving and embroidery designs are very time consuming to make. It is not unusual for traditional textile artists to take several weeks to make a work of their art.

We start with weaving, a textile art that is common in Otomi culture. Weaving is a practice that dates back to pre Columbian time, though some European techniques have been introduced to it over the years. The mathematical aspects of weaving involve a lot of thread counting and keeping track of those thread counts. Conversations I had with traditional Otomi weavers in Tolimán (in the Mexican state of Querétero) in 2001 revealed that in the past Otomi weavers kept track of all thread counts by memory. Jacques Soustelle (1993) [1937] published one of the most complete anthropological studies of Otomi culture and in it he describes his observations of traditional Otomi weavers and embroiderers. On page 94 he states, "La base es la cuenta de hilos. . . . La memoria de los tejedores Otomies es tan perfecta, sus cuentas de hilos tan exactas, que . . . , queda perfectamente simétrico." This translates to "[the art of weaving] is

---

[4] From discussions with Otomi artists and with Prof. Richard Ramsey (personal communication), some Otomi artists now calculate thread counts on paper, and beginning artists sometimes work from a previously finished design. Nevertheless, most experienced textile artists still create their work completely from memory.

based on thread counts. . . . . The memory of the Otomi weavers is so perfect, their thread counts are so exact, that . . . , [the final product] ends up perfectly symmetric." The description by Soustelle indicates that what the Otomi artists are doing does in fact require mathematical processes of organizing information (via the thread counts) and knowledge of concepts of symmetry.

Embroidery is a related textile art for which Otomi artists also typically have substantial skill. In the case of embroidery there is usually more complicated patterns, often including several figures that have properties of symmetry.

Let us look at some specific examples of Otomi embroidery. Figure 4 shows a cloth from an Otomi market in Ixmiquilpan, in the Mexican state of Hidalgo.


Figure 4. *An Otomi cloth. Photograph by the author.*

If we look at the pattern of the bird images that appear in the figure, we can see that they are symmetric about a vertical line that could be drawn down the middle of the figure. Also, they are horizontally symmetric, if we draw an imaginary horizontal line through the middle of the figure.

In fact, the person who embroidered this piece understands the concepts of vertical and horizontal symmetry. Evidence of this comes from observations by anthropologists. I quote (Soustelle, 1993) [1937]) again, p. 94, ". . ., se hace generalmente el motivo de manera que quede situado justo en medio del sarape, . . . , es necesario . . . hacerlo en dos etapas, una mitad sobre el borde derecho de la banda, la otra sobre el izquierdo." This translates to ". . . , in general the pattern is made in such a way as to be situated in the middle of the shawl, . . . , and it is necessary . . . to make it in two stages, half on the right hand border and the other half on the left side." Thus, the artist creates half of the figure, then completes the other half. In order to make the figures symmetric, the artist must create the second half of the figure in the proper orientation. This means the artist must understand *a priori* how the symmetric pattern must look.

As for measurements, a close look at Otomi textile work shows very precise spacing, such as the sizes of figures, spaces between figures, and so on. Nevertheless, the artists create their works without the use of diagrams or rulers. The precise measurements are made from memory and from a lot of practice in the embroidery process. Figures 5 and 6 show other Otomi textile examples. I leave it to the reader to decide which of the symmetry properties (described for example in (Ascher, 1991; Chapter 6) these examples possess.

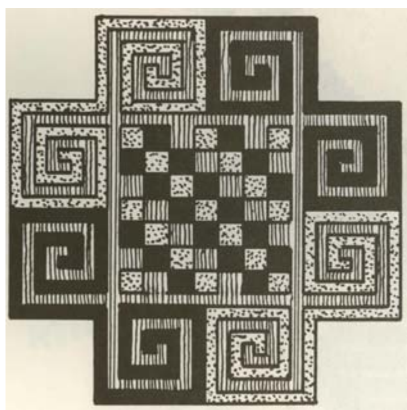

Figure 5. *Otomi pattern from a shoulder cape*. From Irmgard Weitlaner- Johnson, *Mexican Indian Folk Designs, 252 Motifs from Textiles*. Dover Publications, 1976, Figure 64, p. 24.

Figure 6. *Otomi pattern from a shoulder cape*. From Irmgard Weitlaner- Johnson, *Mexican Indian Folk Designs, 252 Motifs from Textiles*. Dover Publications, 1976, Figure 66, p. 25.

**4.** *Otomi mathematics in the Codices*.

Part of the long and complicated history of interaction between the Aztecs and Otomies involves the fact that much of Otomi territory was eventually controlled by the Aztecs which led to ongoing conflict between the two cultures. The Aztecs collected tributes from groups they controlled and descriptions of such tributes appear in various codices relevant to Aztec history. Several such codices contain mathematical information, and we will see that this information tells us something about Otomi mathematics.

First, a description of the mathematical symbols that appear in such codices is shown in Figure 7 below. These symbols appear most often in context of tributes that were given to the Aztecs from people in territory they controlled, and later, in the context of tributes given to local Spanish officials.

*(a)*

*(b)*

*(c)*

*(d)*

Figure 7. *Mathematical symbols from Mesoamerican codices*: (a): The symbol for 1 (units), (b): The symbol for 20, (c ): The symbol for 400, (d): The symbol for 8000. These figures were made by the author, using pages 42 – 56 of (Ross , 1978) as examples.

Figure 8 below shows a folio (page) from the *Mendoza Codex* in which some mathematical symbols appear. This codex was written by a person of Aztec culture, shortly after the Spanish conquest of Mexico[5] (see Ross, 1978; p. 11).

[5] From 1519 to 1521.

Figure 8. *A folio from the Mendoza Codex.* To the left are the glyphs for Mixquiahuala and Ixmiquilpan. This image is from: *Mendoza Codex*, (1978), with Comments by Kurt Ross, Figure (F), p. 46.

Several folios of the codex describe tributes the Aztecs collected from towns and locations they controled. There is handwriting in several places. The handwriting was done by a Spanish person who gave explanations for the images. Observe the symbols for 20 and 400 in several places. Along the left side of the folio there are ten drawings with writing above each one. These drawings are glyphs that identify which towns and locations had delivered tributes that are described on that folio. The third and fourth glyphs from the bottom are those of Ixmiquilpan and Mixquiahuala[6], respectively. These two locations are in the Mezquital Valley, and have been considered part of Otomi territory since before the *Codex Mendoza* was written. Thus, in this folio some of the tributes were delivered by Otomies to the Aztecs. It is reasonable to conclude that the Otomies that were involved with that delivery must have understood the mathematical symbols that were used to represent the quantities.

[6] These glyphs are known to be those of Ixmiquilpan and Mixquiahuala, however if you look up the work by Ross and magnify those images, it is possible to read the words "Ixmiquilpan" and "Mixquiahuala" directly.

Below I will describe another codex that we could use as a comparison, but first, some more details about the folio from the *Mendoza Codex* are in order. Notice the images near the top of the folio. They are square- looking images with a symbol  of 400 above each one. In fact, these images represent textiles that were given to the Aztecs as part of the tributes they collected. According to (Ross, 1978: p. 37), the icons with colored patterns in them represent such things as mantles and tunics that have some pattern to them. Ross explains (ibid, p. 46) that such textiles were "richly embroidered." We can interpret "richly embroidered" to mean that the textiles contain several patterns. As we know from the discussion of Otomi art, this means the textiles likely included properties of various kinds of symmetry, and took a long time to make. The blank icons represent larger mantles without special patterns. There are a total of 6 x 400 = 2400 total textiles that were delivered as tributes by the ten locations to the left. The images below the textiles represent war suits (also called war dresses) and shields. On page 37 of (Ross, 1978), he tells us that tributes of mantles were collected every six months and the war suits every year. Two such images have the symbol  for 20 connected to them, so more than 40 such suits were delivered to the Aztecs. From the previous section we know that creating these textile products is a complicated process that includes mathematical concepts and thinking. Also, making more than 2420 mantles, tunics, war dresses, and shields represents a lot of work by many people! The fact that the Aztecs collected them shows that they had an appreciation of the work involved in making them, and probably held the textile artists in high regard.

Figure 9 below shows a folio from a fragmented codex known as the *Mixquiahuala Codex*. The most recent and complete description of the *Mixquiahuala Codex* that I know of is (Hermann Lejarazu, 2001).

Figure 9. *The Humbolt* 7 *Fragment*, from the *Mixquiahuala Codex.*

On page 96 of Hermann Lejarazu's article he explains that the page known as the *Humbolt* 7 *Fragment* that appears in Figure 9 was written in about 1571. Above each drawing there we can see either some small circles, one or more flags, or a larger circle that may have a small circle drawn inside it. The larger circles with (in some cases without) smaller circles within them represent the amount in tomines (the local currency used at the time) that the local Spanish authority paid for the quantities of goods delivered. The figures immediately above each drawing is represented by either the symbol for one, or flags for twenty. As an example, the last row of figures represents (from left to right) four bundles of firewood, four fish, 20 baskets of tortillas, another 20 baskets of tortillas and four more fish. The symbol at the end of that line describes an approximate date of delivery in the form of a representation of the moon. What distinguishes this fragment from the folio of the *Mendoza Codex* of Figure 8 is that the *Humbolt* 7 *Fragment* represents an Otomi- Spanish communication. We can see that the pre- Hispanic symbols were still in use at time of the writing of this fragment. This gives us more confidence to conclude that the mathematical symbols for one, twenty, 400 and 8000 were known to and understood by people of Otomi culture. There are more examples of the use of these mathematical symbols by people of Otomi culture, such as in the *Osuna Codex*; see (Valle, 2001).

**5.** *The Otomi calendar*.

The Otomies used a calendar system that consisted of a 365 solar calendar and a 260 day ritual calendar. This structure is similar to many Mesoamerican calendars; see (Broda de Casas, 1969). Our discussion of the Otomi calendar here can be modified for consideration of some other Mesoamerican cultures such as the Aztecs or Mayans. First I will describe the basic structure of the Otomi calendar and later consider the mathematical aspects of Leap Year, and the implications of this question to interpreting the mathematics of the Otomi.

The primary source for information on the Otomi calendar comes from the *Huichapan* (pronounced "wee- CHAH- pahn") *Codex*. The *Huichapan Codex* was probably written sometime shortly after 1632, according to (Caso, 1967; p. 211). It is a post Conquest document that is written in Otomi with Spanish translations written near the Otomi text. Many anthropologists have studied it and have agreed that it was originally written by a person of Otomi culture.

The structure[7] of the Otomi solar calendar is that of 18 cycles of 20 days each for a total of 360 days, followed by 5 extra days added at the end for a total of 365. The ritual calendar is calculated as 13x20 = 260 days. The Otomi expression for a given year[8] appears as a drawing that consists of one to thirteen small circles placed near a glyph that represents one of twenty day names. Figure 10 below shows two such dates from the *Huichapan Codex*.

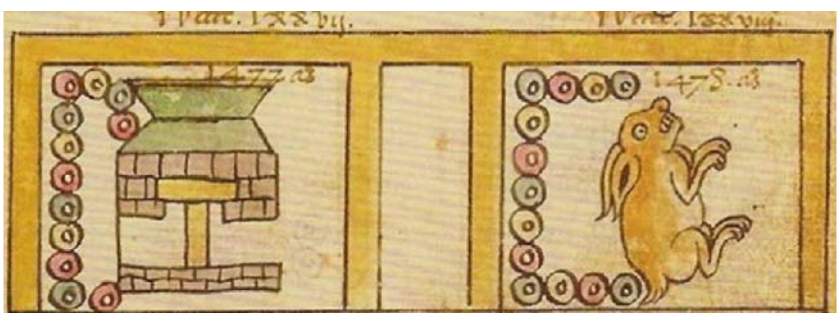

Figure 10. *Two Otomi calendar dates from the Huichapan Codex.*

From left to right, the two dates shown above are: 12 *House* and 13 *Rabbit*. We will see shortly that the explanation for using numerical values of one to thirteen and twenty day names is that by counting cycles of 365 days in this manner, it is easy to predict when the 260 day ritual

[7] The structure of the Otomi calendar follows a pattern similar to calendars used by many Mesoamerican cultural groups. There are unresolved questions about how the18 x 20 + 5 structure made its way across Mesoamerican cultures. Because the Otomies are a very old cultural group, further information about how they created their calendar system might lead to more insights into this issue.
[8] The Otomi description of years is also similar to that of other Mesoamerican calendars.

calendar and 365 day solar calendar coincide. That coincidence occurs when the least common multiple (lcm) of both calendar cycles agree, which is:

$$\text{lcm}(365, 260) = \text{lcm}(73\text{x}5, 13\text{x}20) = 13\text{x}73\text{x}20 = 18{,}980 \text{ days} = 365\text{x}52 = 260\text{x}73.$$

Thus, every 52 solar years the two calendars coincide.

Now let us look at the description of the Otomi years. Table 11 shows the list of all twenty day names used by the Otomies in their calendar.

| *Day* | *Name* | *Day* | *Name* |
|---|---|---|---|
| **1** | Knife | **11** | Monkey |
| **2** | Wind | **12** | Yellow herb |
| **3** | House | **13** | Cane (sugar) |
| **4** | Lizard | **14** | Bite |
| **5** | Snake | **15** | Eagle |
| **6** | Death | **16** | Turkey |
| **7** | Deer | **17** | Earth movement |
| **8** | Rabbit | **18** | Flint |
| **9** | Water | **19** | Rain |
| **10** | Dog | **20** | Flower |

Table 11. *The twenty day names of the Otomi calendar*. Translated from (Soustelle, 1993 [1937]; p. 519).

In descriptions of Otomi years, such as in the *Huichapan Codex*, there are only four day names that are ever used to describe years: *House, Rabbit, Cane*, and *Flint*. Why only four? The explanation is that for 365 days, we have 365 = 360 + 5 = 18x20 + 5, so after five cycles of 20, there are five more days that are named. Because 20/5 = 4, it leaves only four choices for the name of the day at which the new year begins. Observe that thi way of using four day names to denote years and up to thirteen numerical symbols means that the solar and ritual calendars will coincide when the first pattern repeats; that is, when the thirteen number symbols have cycled four times for a total of 52 solar years.

Now it is time to consider the question of Leap Year in the Otomi calendar. Because of the proximity of the writing of this codex and the declaration in Europe of the Gregorian calendar (by Pope Gregory VIII in 1582), there was likely a strong interest in whether calendars of indigenous groups of the Americas had made adjustments to their calendars to account for what Western calendars refer to as Leap Year: The addition of an extra day every four years, with minor adjustments. In the case of the Otomi calendar, the author of the *Huichapan Codex* states in Otomi on folio 13: (Lastra y Bartholomew, 2001; p. 43):
**“***N˜uccãdaandaqhueya. Edettatemahiãntitemaquüttamapa. ccclxv yquüttzi edato oras. ãnãbeattegui. Nãh˜u oras. Emãh˜equi e oras.”* This statement is followed by written comments in Spanish: *“Cada año [consta de] dieciocho veintanas más cinco días [= 365] ccclxv . . se añaden seis horas [del] reloj . . ., se llaman horas iguales horas.”*

The above translates to an explanation of a calculation of Leap Year by adding six hours to each year, the total over four years giving 6x4 = 24 hours. This would seem to imply a calculation of Leap Year in the Otomi calendar. Although there seems to be some logic in this description, from a cultural- mathematical point of view, we are led to doubt it[9]. The first problem is that there is no evidence that the Otomi divided their days into smaller units (“hours”). Although this is certainly possible (e.g., by observing shadows on a fixed object), the Otomies would not have made a calculation of 6 x 4 = 24 in this context. Indeed, we have seen that the Otomies use a 5- 10- 20 number system in which 24 would not be a natural choice.

The fact that only four names are used to describe years also tells us something about the possibility of adding a day to calendar after every four years, as would be done to include Leap Year as it is known in Western terms. In fact, adding one more day would break the pattern of using only four day names to describe years. For example, if at the end of a year 7 *Cane* there would be a Leap Year calculation, the following year would then be named 8 *Rain* because of the

[9] Jacques Soustelle (1993 [1937]; p. 520) raises similar doubts in his discussion of the Otomi calendar. Our observations here are mainly on the mathematical implications of making such a calculation.

addition of one day. However, as far as we know the Otomies never made this change in their descriptions of years.

Our final conclusion of the 6x4 = 24 statement that appears in the *Huichapan Codex* is that the Otomies did not measure Leap Year this way. The question then becomes: Did the Otomies account for Leap Year in some other way? We must be careful not to conclude that just because a culture does not make a Western style calculation, it does not imply that the culture is ignorant of the relevant mathematics. It is widely known that many Mesoamerican cultures were very knowledgeable of astronomy and cycles of the sun, the moon, Venus, and other astronomical phenomena such as equinoxes and solstices. Moreover, the Otomi calendar was very closely connected to agricultural cycles, as described for example in (Albores, 2006). Hence, in order to maintain their calendar as an accurate tool for agriculture, the Otomies must have adjusted it from time to time. How the adjustments were made is not known. This question is addressed to some extent in (Broda de Casas, 1969), and Beatriz Albores describes a possibility that some Mesoamerican cultures related to the Otomies may have used the cycle of Venus to adjust their solar calendar; see (Albores, 2006).

ACKNOWLEDGEMENTS: I would like to acknowledge research support for this paper as part of a Fulbright García Robles Scholarship at the Instituto Tecnológico Autónomo de México (ITAM), Mexico City, during the 2006- 2007 school year. I am also grateful to the University of North Dakota for Developmental Leave support during that time as well. Finally, I would like to thank the people who participate in the annual Coloquio Otopame meetings in Mexico, and those who participate in the Otopame seminar at the National University of Mexico (UNAM) in the Instituto de Investigaciones Antropológicas (Institute of Anthropological Studies). The many discussions regarding Otomi culture have been very helpful to me in preparing this paper.

## References


Acuña, René.
1990 *Arte breve de la lengua Otomi y vocabulario trilingüe, Español- Náhuatl- Otomi, de Alonso Urbano, 1605*. Reprinted by the National University of Mexico (UNAM), Mexico City.

Albores, Beatriz.
2006 *Los graniceros y el tiempo cósmico en la región que ocupó el Matlanzinco*. In *Estudios de Cultura Otopame*, vol. **5**. Mexico City: Instituto de Investigaciones Antropológicas, UNAM, 71 - 117.

Ascher, Marcia.
1991 *Ethnomathematics: A Multicultural View of Mathematical Ideas*. Pacific Grove: Brooks/Cole.

Barriga Puente, Francisco.
1998 *Los Sistemas de Numeración Indoamericanos; un enfoque areotipológico*. Mexico City: UNAM.

Bartholomew, Doris.
2000 *Intercambio lingüístico entre Otomi y Náhuatl*. In *Estudios de Cultura Otopame*, vol. **2**, Y. Lastra and N. Quezada, Eds. Mexico City: UNAM, 189 - 201.

Broda de Casas, Johanna.
1969 *The Mexican calendar as compared to other Mesoamerican systems*. Acta Ethnologica et Linguistica, Series Americana, vol. **4**, Wien: Institut für Völkerkunde der Universität Wien.

Caso, Alfonso.
1967 *Los Calendarios Prehispánicos*. Mexico City: UNAM, Institutode Investigaciones Históricas.

Closs, Michael P., Ed.
1986 *Native American Mathematics*. Austin: University of Texas Press.

Gilsdorf, Thomas E.
2008 *Etnomatemáticas de los Otomíes*. In *Estudios de Cultura Otopame*. Vol. **6**, Mexico City: UNAM, 165 - 181.

Hermann Lejarazu, Manuel.
2001 *Códices tributaries de Mezquiahuala*. In *Códices del Estado de Hidalgo, State of Hidalgo Codices*. Laura Elena Sotelo Santos, Victor Manuel Ballesteros García, Evaristo Luvián Torres, Eds. Pachuca: UAEH, 88 – 99.

Lastra, Yolanda.

2006 *Los Otomies, su Lengua y su Historia*. Mexico City: Instituto de Investigaciones Antropológicas, UNAM.

Lastra, Yolanda, y Bartholomew, Doris, Eds.
2001 *Códice de Huichapan. Paleografía y traducción* (of notes by Lawrence Ecker). Mexico City: Institutode Investigaciones Antropológicas, UNAM.

Merrifield, William R.
1968 *Number names in four languages of Mexico*. In *Grammars for Number Names*, H. Brandt Corstius, Ed. Dordrecht: D. Reídle Publishing Company.

Ross, Kurt.
1978 *Codex Mendoza, Aztec Manuscript*. Miller Graphics Liber S. A., CH-Fribourg.

Smith, Michael.
2003 *The Aztecs, Second Edition*. Malden: Blackwell Publishers.

Soustelle, Jacques.
1993 *La famille Otomi – pame du Mexique Central*, Travaux en Mémoires de l'Institut d'Ethnologie, Paris, 1937. Translation into Spanish: *Centro de Estudios Mexicanos y Centroamericano*s, Fondo de Cultura Económica, 1993.

Thomas, Cyrus.
1897 *Numeral systems of Mexico and Central America*. In *Nineteenth Annual Report*, Bureau of American Ethnology. Washington, DC.

Valiñas Coalla, Leopoldo.
2000 *Lo que la lingüística yutoazteca podría aportar en la reconstrucción histórica del Norte de México*. In *Nómada y Sedentario en el Norte de México, Homenaje a Beatriz Branniff.* Marie- Areti Hers, José Luís Mirafuentes, María de los Dolores Soto, Miguel Vallebueno, Eds. Mexico City: UNAM, 175 – 205.

Valle, Perla.
2001 *Códice Osuna*. In *Códices del Estado de Hidalgo, State of Hidalgo Codices*. Laura Elena Sotelo Santos, Victor Manuel Ballesteros García, Evaristo Luvián Torres, Eds. Pachuca: UAEH, 56 – 63.

Weitlaner- Johnson, Irmgard.
1976 *Mexican Indian Folk Designs, 252 Motifs from Textiles*. Mineola: Dover Publications.

Wright, David.
1997 *Manuscritos Otomies del Virreinato*. In *Códices y Documentos sobre México, Segundo Simposio, Volumen II*, Rueda Mithers, Salvador, Vega Sosa, Constanza, and Martínez Barca, Rodrigo, Eds., Mexico City: InstitutoNacional de Antropología e Historia (INAH), 437 – 462.

Wright, David.
2005 *Hñahñu, Ñuhu, Ñhato, Ñuhmu*. In *Arqueología Mexicana*, vol. **13**, No. 73, May - June, 19.