Preface

Regardless of how you view mathematics, whether you like it or not, whether you feel you could explain what "mathematics" or not, most people would agree that mathematics is a way of communicating information. That information may seem quite different than information that is communicated orally or in writing, yet it is still information of some kind. As a form of communication, mathematics is subject to influences and variations in interpretation both at the societal/cultural level, and even at the individual level. In other words, our understanding of what is or is not mathematics is influenced both by what is perceived to be mathematics by the culture we live in, and by what is perceived to mathematics by each person in that culture. The interaction between mathematics and culture is what this book is all about. My choice of the expression "cultural mathematics" in the title comes from my intention that prospective readers know right from the start that the two main subjects are mathematics, and culture. This topic also goes by other names that you may have heard about or read about. Cultural mathematics is also called *ethnomathematics*, a term first defined by Ubiratan D'Ambrosio in the 1980s. D'Ambrosio is considered to be the person who made the study of the interaction between mathematics and culture into an established discipline of study. Other expressions associated with the study of mathematics and culture include *multicultural mathematics*, *traditional mathematics, indigenous mathematics*, and *oral and written mathematics*.

My goal in writing this book is to make available a resource that could serve as a textbook for courses in which there is a significant emphasis on the connections between culture and mathematics. Although there are many excellent sources of information on the cultural aspects of mathematics, my idea is to include some course- like structure with my exposition, such as exercises and topics for essay/discussion. I have attempted to write this for use as the main text for a course on cultural mathematics (or ethnomathematics), or as a main or supplementary text for courses in: mathematics education, multicultural education, graduate courses on the connections between education and culture, for use as a supplementary text for courses in the history of mathematics, or as a general reference for anyone who is interested in the connections between mathematics and culture. This book is roughly at the level of someone who has had college algebra, however, the most important prerequisite for a reader is an interest in discovering connections between mathematics and culture.

I believe the beauty of mathematics should be enjoyed because of the human creativity that went into making it. After all, mathematics is very much a human endeavor. Unfortunately, much of modern mathematics is presented in a way that disconnects it from the fact that it has been created by people. What makes cultural mathematics so interesting to me is that the human aspect of it is *unavoidable* and hence, plays a central role. The mathematics does not have to be confusing or esoteric to be beautiful. One of my main goals has been to maintain this attitude throughout the book.

For those who seek cultural mathematics that does require deeper (Western) mathematical experience, I have included such mathematics in some of the exercises marked with an asterisk (*), and in the notes at the end of each chapter I have sometimes indicated where a person can investigate topics further.

Writing about the topic of cultural mathematics for readers with backgrounds primarily in Western mathematics brings one to a dilemma: On one hand, using Western terminology and notation to describe mathematics of non- Western cultures is inherently inaccurate because people in such cultures would not think of the mathematical content in the same way as it is perceived in Western culture. On the other hand, if the goal is for people of Western backgrounds to understand how cultural activities can be understood as mathematics, then one must speak to readers in familiar mathematical terms.

Thus, it is crucial that the reader have a foundation that includes an understanding what "culture" means, what constitutes "mathematics", how a person's culture affects and often blurs that person's perceptions of mathematics, and how to keep aware of cultural tendencies and perceptions as one tries to understand the mathematics of other cultures. The goal of Chapter One is to put together such a foundation. In my effort to create it, I have rewritten Chapter One several times. I have numerous "half baked" word processing files to prove it! The final, "fully baked" version of Chapter One is much longer than I first imagined it to be but that chapter is, without question, the most important one of the book.

I would like to briefly discuss some peripheral considerations that are beyond the intentions of this book. One such topic consists of the political and social aspects of cultural mathematics. These two aspects have to do with how a culture's political or social situation affects perceptions (by all cultures involved) of mathematics, how political and/or social events have affected (often adversely) the preservation of cultural knowledge, including mathematical knowledge, and the role mathematics (or science) plays in social/political contexts. Examples could include such things as the effects on mathematics of one culture ruling over another culture, or DNA testing to determine cultural backgrounds, (plus the mathematics related to such contexts). I believe these political and social aspects of mathematics are important considerations that relate to culture, however, I am not an expert in either of these areas, and so I have not tried to address them here. On the other hand, I hope this book will be useful to those who do have more expertise in these areas, and it would be great if some of those readers would be motivated to create scholarly works in the areas of social and political aspects of cultural mathematics.

The structure of the book is as follows: Part One, which consists of Chapters One through Seven, is a description of the general ideas of cultural mathematics. Part Two, consisting of Chapters Eight and Nine, represents two "case studies", in which I apply the ideas of Part One to a specific culture. The two cultures I chose, the Otomies of Mexico and the Incas of South America, are the two cultural groups I know the best in terms of

cultural mathematics. At the end of each chapter I have put exercises, followed by further notes about the topics of that chapter.

About the exercises. I have divided them into three categories. The first type of exercise is **short answer**. In these questions I try to prod the reader to contemplate ideas related to the topic at hand. Second are **calculations**, in which the reader can practice the relevant concepts. In some cases the process is similar to how people in the cultural group at hand would have solved the problem. In other cases the process has been restated in Western mathematical terms. Working through the calculations this way gives the reader a feel for how the calculation is accomplished. The third type of exercise is **essay/discussion**, in which the reader can delve more deeply into a topic, or could serve as topics for group discussions. Whenever an exercise requires some specialized mathematical knowledge (e.g., at or beyond the level of calculus), I have alerted the reader to that by marking the exercise with an asterisk (*).

At the end of the book there is a bibliography, in which I have attempted to include as many references as I could think of so that interested readers can seek more information and/or study further.

Also at the end of the book, there are hints and comments about some of the exercises. I chose not to simply put in numerical answers to some problems for essentially two reasons. One reason is that I do not believe in the statement, “If I get the

correct answer, then I understand the concept." A second reason is that a course in cultural mathematics, being very different than a typical mathematical course, requires a broad state of thinking. There are many situations in which more than one solution or explanation is possible. There are others in which we do not know what the solution or explanation is. In some cases, I simply ask the reader to describe one of perhaps many possible explanations to the situation or concept being considered. On the other hand, I want readers to feel that they understand what the topic is about. Hopefully, the suggestions and comments will lead readers toward a better understanding of the topic relevant to the exercise.

On using this book in a classroom.

If you are going to use this book as a text for a course, I have some comments about its organization and such. Perhaps this is even the first time you will teach a course in cultural mathematics. In any case, I think you will find that it is quite different than teaching a traditional mathematics course.

Motivational questions: One of the most exciting aspects of learning about cultural mathematics is that a person gets to think about mathematics in a very creative way. In my own teaching experience, I have found that students start thinking creatively about mathematics if I ask them to describe their own interpretation of concepts before going through the material. The motivational questions at the beginning of each chapter

are intended to mimic this. My suggestion to you is to have the students explain verbally or write down on paper, their responses to the motivational questions before you begin to go through the topics. If you feel that some students will not put much effort into responding to the questions, you could have their responses to the questions be worth a nominal number of points. Also, you may wish to mention to them that their responses should be based on how they perceive the topic, not on how they think it is "supposed to be". You will notice that during the rest of a particular chapter I discuss the issues that come up regarding the motivational questions, however the intention of the questions is to have students think carefully about certain aspects of mathematics as they apply to their personal lives and experiences, and then, during the rest of the chapter, they can see how people who have other experiences view the same mathematical aspects.

The classroom setting: Another exciting aspect of learning about cultural mathematics is that it is an excellent opportunity to *discover* mathematical concepts. In order to set up that kind of environment, I have written the book in an informal style, with the idea that the classroom setting will be one in which everyone feels encouraged to think about mathematics and discover mathematical ideas, including *you*, the instructor. Below are some aspects of teaching a cultural mathematics course that I believe will enhance the experience for you and for your students. My intention has been for this book to make the aspects easier to implement:

• Have students draw on their own experiences from their personal lives (such as family life and background, job or school experiences, previous mathematical experiences, and so on).

• There are many ways to interpret how humans do mathematics, so keep in mind that the course is not about finding "correct answers". Rather, it is about realizing that human diversity manifests itself in mathematics just like it does in many other disciplines.

• As often as possible, try to have students discover the mathematics on their own. For example, most of the exercises can be given to students in class, the idea being that they work in groups or individually to come up with a response. Those responses could either be turned in and counted for points, or discussed in class. You could have some exercises done in class and others done as assignments, for example.

• It is not necessary for you to be an expert on the topics covered in this book in order to teach a course in cultural mathematics. In fact, you should expect that from time to time students will ask about or comment on something for which you do not have full information (this happens to me frequently, whenever I teach cultural mathematics). Such moments represent opportunities for students to learn and discover on their own. You could ask the students, or better yet, the whole class, to look up references in the bibliography of this book, go to the library, arrange to interview someone related to the topic, and so forth, in order to understand the topic.

Suggested chapters to cover: The table below may be helpful in determining which chapters to go through in using this book in a course.

Intended use of the text	***Suggested Chapters***
Main text for a two semester credit course	1 - 3, *at least two others, plus either Chapter 8 or 9*
Main text for a three semester credit course	*Most of the book*
Supplementary text	1 - 3, *at least one other chapter*

Regardless of how you use this book, Chapter One is essential reading. I think I have hinted at this already! It sets up how to think about mathematics in a cultural setting

Here are a few questions that may come up regarding teaching a course in cultural mathematics (and using this book):

- *What if a student starts describing mathematics of a culture I know nothing about?*

This kind of situation is something to look forward to. Really. My first comment is to repeat my previous remark that no one, including you the instructor, is expected to be an expert at everything. So, when someone begins describing a culture you do not know about, it is a great opportunity for that student to become an expert on that particular culture. In other words, instead of having you bear the responsibility of learning about the

culture and explaining it to the class, it should be the students who do the learning and explaining. What I have done in my experience is the following:

1) Include some unspecified activities in your course syllabus where students have some choice of topics. Something like a short essay (1 - 3 pages), or a class presentation (either individually or in groups) has worked well for me. That way, you can respond to a student's comments on a culture (or topic) that you are not familiar with by saying something like, "That sounds very interesting. On one of the essays (or class presentations) you can choose that as your topic."

2) It is important to have accountability in item 1) above. You should require that students include a minimal number of formal sources (i.e., published books or journal articles) in their essay or presentation. The bibliography in this book has a pretty large list of sources I am familiar with, but if you want to make sure students access as many sources as possible, it might be worth your while to either talk with a librarian at your school, or arrange for that person to go through the process of looking for resources (either in your class or at the library)

- *What if a student asks a mathematical question I can't answer?* This is another good "teaching moment." It could easily happen that a student asks a curious question for which the mathematical answer is quite difficult to explain. For example, in the chapters on games and calendars, there are questions that can come up whose explanations quickly go beyond the level intended in this book. So, what to do? First, if you feel the topic is

definitely beyond the scope of your course, you should probably just say so. *However*, do not end the discussion there. You can refer the student to the bibliography in this book where there are some mathematical references, or you can refer the student to a mathematician who you think could explain the concepts to the student. If you know a mathematician whose expertise could be useful in your class, you may consider inviting her or him to your class to explain some mathematical details.

• *Won't students think the course material is "watered down" because memorizing formulas and theorems are not emphasized?* Gosh, I hope not. However some might think this anyway. In my experience I have found that it is best to maintain an activity ratio of about one third between the short answer, calculations, and essay/discussion activities. Students who have strong backgrounds in calculating activities may find such activities easier, however, they will still have to participate in short answer (connecting concepts), and discussion/essay activities, both of which require distinct types of thinking.

• *What if I fall so far behind schedule that I will not be able to cover all topics?* In every course I teach a strive for *quality* over quantity. If you are behind your schedule, you can start choosing the topics you consider most crucial to the goals of your course. In most chapter the early discussions and examples tend to be less time-consuming, and you can cover such earlier parts and plan to come back later for more depth if time permits. At any rate, you should *not* feel obligated to race through the material just to say you

finished it. Another point to consider is that this book contains somewhat more (though hopefully not excessively so) material than what one would cover in a typical course. The reason for this is not to make instructors feel obligated to cover everything, rather, it is to give instructors some flexibility in how to run their own courses. For example, if in your course you want to spend a lot of time discussing calendars, then you will find plenty of material in Chapter Seven on that topic. Emphasizing that chapter will probably mean you will have less time for other chapters, but that flexibility is up to you.

This work grew out of my experiences in cultural mathematics. Some of that experience includes courses in ethnomathematics that I have taught at the University of North Dakota, and at the Instituto Tecnológico Autónomo de México (ITAM), and from giving presentations on topics within cultural mathematics at several universities and conferences. The rest of my experience comes from about twelve years of exciting experiences in learning and researching various aspects of cultural mathematics. I have been inspired by a few specific works, namely (Ascher, 1991), (Ascher, 2002), (Zaslavsky, 1999), and (Closs, 1986). As a reader, you will notice the influence of those references in this book. Finally, although I have tried to include examples of cultures from as many parts of the world as possible, there is somewhat more material about cultures in the Americas. This is because most of my experience is with cultures in this geographical area.

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All diagrams, drawings, and photographs that are not cited from a specific reference were made by me.

Thomas E. Gilsdorf Grand Forks, April, 2009